Renormalization of the NN interaction: one boson exchange

Álvaro Calle Cordón
(In colaboration with E. Ruiz Arriola)

University of Granada, Spain.

MENU 2010, Williamsburg, VA, USA
May 31, 2010

Outline of the talk

Outline of the talk

MOTIVATION: NEED FOR RENORMALIZATION

- Nuclear force at the hadronic level

  $\Rightarrow$ unknown at short distances (high momentum) $\Leftarrow$
- Its non-perturbative nature

  $\Rightarrow$ better handled with quantum mechanical potentials $\Leftarrow$
- Chiral symmetry

  $\Rightarrow$ NN forces of practical interest in nuclear physics $\Leftarrow$
- Chiral expansions $\Rightarrow$ singular potentials at short distances

$$V(r) \to \pm\frac{1}{r^n}, \text{ for } r \to 0 \text{ and } n > 2$$

Renormalization is the most natural tool to handle singularities.

MOTIVATION: CHIRAL POTENTIALS

- Coordinate space renormalization has been used for chiral and singular potentials (Pavón+Arriola,PRC74:054001,2006.)

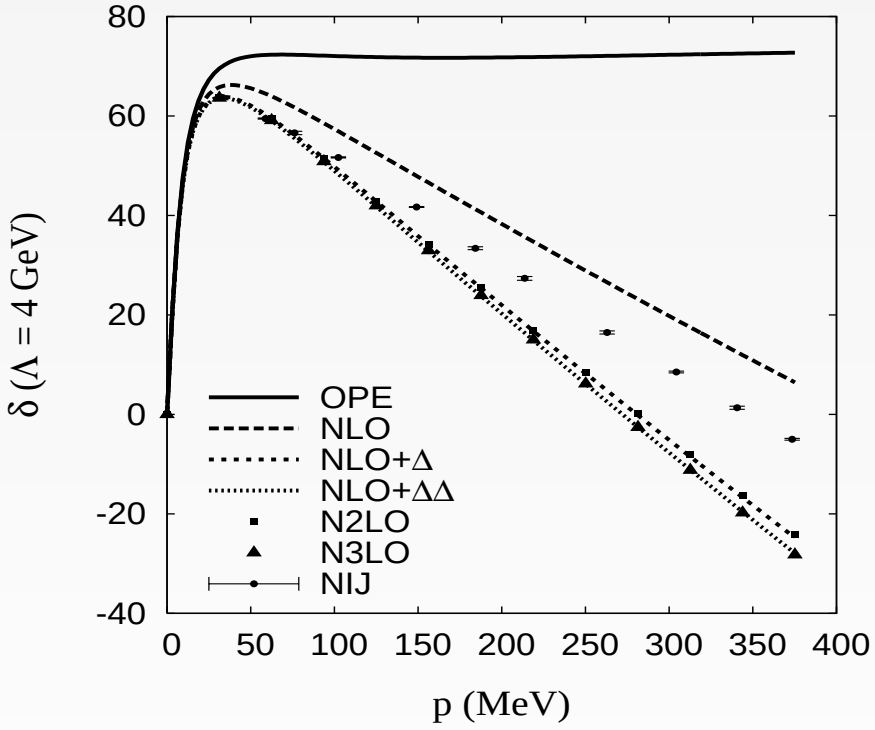
$$V(r) \to \frac{1}{f_\pi^n M_N^m} \frac{1}{r^{n+m+1}}$$

- Results converge for practical cut-offs $r_c \sim 0.5\mathrm{fm}$ which is $\sim 1/p_{\max}$. The more singular the better.

- Renormalization with a BC is equivalent to put counterterms in the Lippmann-Schwinger equation in momentum space but computationally more efficient (Entem+Pavon+Machleidt+Arriola, PRC77:044006,2008).

- TPE relativistic potentials (Higa+Pavón+Arriola, C77:034003,2008.)
- TPE with Δ's (Pavón+Arriola, PRC74:054001,2006.)

MOTIVATION: CHIRAL POTENTIALS ($^1S_0$ PHASE SHIFT)

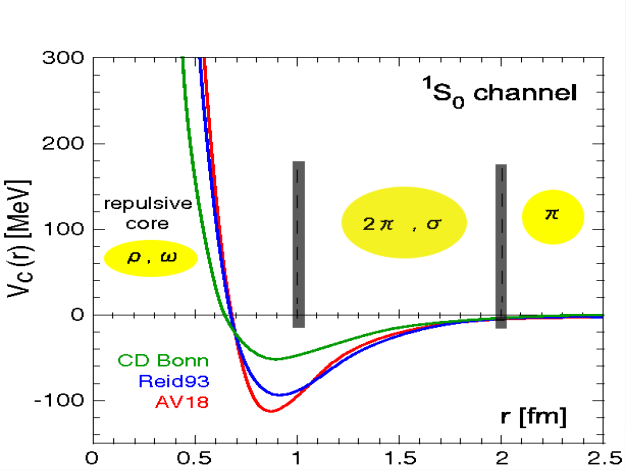


In all TPE approaches there is overbinding $\sim 5 - 10^0$.
Missing physics σ, ρ, ω *?.*

Outline of the talk

OBE model of the NN interaction

- Includes all mesons with masses around the nucleon mass, i.e., π, σ, $\rho(770)$ and $\omega(782)$



$$\mathcal{L}_{\pi NN} = -\frac{g_{\pi NN}}{2\Lambda_N}\bar{N}\gamma_\mu\gamma_5\tau\cdot\partial^\mu\pi N\,,$$

$$\mathcal{L}_{\sigma NN} = -g_{\sigma NN}\,\sigma\bar{N}N\,,$$

$$\mathcal{L}_{\rho NN} = -g_{\rho NN}\bar{N}\tau\cdot\rho^\mu\gamma_\mu N - \frac{f_{\rho NN}}{2\Lambda_N}\bar{N}\sigma_{\mu\nu}\tau\cdot\partial^\mu\rho^\nu N$$

$$\mathcal{L}_{\omega NN} = -g_{\omega NN}\bar{N}\gamma_\mu\omega^\mu N - \frac{f_{\omega NN}}{2\Lambda_N}\bar{N}\sigma_{\mu\nu}\partial^\mu\omega^\nu N$$

- In general we get a non-relativistic potential with
  - $\rightarrow$ central and tensor terms,
  - $\rightarrow$ spin-orbit ($\mathbf{L}\cdot\mathbf{S}$),
  - $\rightarrow$ non-local terms (∇^2), ...

The Large N_c expansion (t'Hoof, Witten)

- 70's t'Hoof and Witten clever idea

  Change QCD $SU_c(3)$ to a $SU(N_c)$ gauge group and choose an expansion in $1/N_c$ ($N_c \to \infty$ keeping $\alpha_s N_c$ fix)

- Hadronic spectrum (baryons and mesons are stable)

$$m_{meson} \sim N_c^0,\ \Gamma_{meson} \sim 1/N_c,\ m_{N,\Delta} \sim N_c,\ \Gamma_\Delta \sim 1/N_c$$

- Scattering

$$g_{MMM} \sim 1/\sqrt{N_c},\ g_{k-mesons} \sim 1/N_c^{(k-2)/2},\ g_{MBB} \sim \sqrt{N_c}$$

- If nucleons are heavy baryons in this limit ($m_N \sim N_c$)

  $\Rightarrow$ a smooth limit of the large N_c is possible (Witten,1997)
  $\Rightarrow$ the non-relativistic potential is a well defined

Large N_c potentials (Kaplan, Savage, Manohar)

Spin-flavour structure of the NN interaction:

$$V(r) = V_C(r) + (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) W_S(r) + (\tau_1 \cdot \tau_2) W_T(r) S_{12} \sim N_c$$

Leading N_c - OBEP
not complete large N_c calculation !
not a $1/M_N$ or p expansion !

$$\begin{aligned}
V_C(r) &= -\frac{g^2_{\sigma NN}}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g^2_{\omega NN}}{4\pi}\frac{e^{-m_\omega r}}{r}, \\
W_S(r) &= \frac{1}{12}\frac{g^2_{\pi NN}}{4\pi}\frac{m^2_\pi}{\Lambda^2_N}\frac{e^{-m_\pi r}}{r} + \frac{1}{6}\frac{f^2_{\rho NN}}{4\pi}\frac{m^2_\rho}{\Lambda^2_N}\frac{e^{-m_\rho r}}{r}, \\
W_T(r) &= \frac{1}{12}\frac{g^2_{\pi NN}}{4\pi}\frac{m^2_\pi}{\Lambda^2_N}\frac{e^{-m_\pi r}}{r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right] \\
&\quad - \frac{1}{12}\frac{f^2_{\rho NN}}{4\pi}\frac{m^2_\rho}{\Lambda^2_N}\frac{e^{-m_\rho r}}{r}\left[1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right],
\end{aligned}$$

Corrections $\sim 1/N_c$
– spin-orbit,
– non-local, relativistic,
– meson widths,

Observables accuracy
$\sim 1/N_c^2 \sim 10\%$!

The large N_c OBE potential (S-waves)

Natural values of couplings

- $g_{\pi NN} = 13.1 \sim g_A M_N / f_\pi$ (Goldberger-Treiman π's),
- $g_{\sigma NN} = 10.1 \sim M_N / f_\pi$ (Goldberger-Treiman σ's),
- $g_{\omega NN} = 9 \sim 3\, g_{\rho NN}$ ($SU(3)$ Symmetry + OZI rule)
- $f_{\rho NN} = 15 - 17 \sim (\mu_p - \mu_n - 1)\, g_{\rho NN}$ (VMD)

- $m_\rho = m_\omega \equiv m_v$ strong correlations, we define

$$g^*_{\omega NN} = \sqrt{g^2_{\omega NN} - \frac{f^2_{\rho NN} m^2_\rho}{2M^2_N}}$$

- Natural values imply $g^*_{\omega NN} = 0 - 7$

The large N_c OBE potential (S-waves)

Natural values of couplings

- $g_{\pi NN} = 13.1 \sim g_A M_N / f_\pi$ (Goldberger-Treiman π's),
- $g_{\sigma NN} = 10.1 \sim M_N / f_\pi$ (Goldberger-Treiman σ's),
- $g_{\omega NN} = 9 \sim 3\, g_{\rho NN}$ ($SU(3)$ Symmetry + OZI rule)
- $f_{\rho NN} = 15 - 17 \sim (\mu_p - \mu_n - 1)\, g_{\rho NN}$ (VMD)

- The $^1S_0$ potential becomes

$$V_s(r) = V_t(r) = -\frac{g_{\pi NN}^2 m_\pi^2}{16\pi M_N^2} \frac{e^{-m_\pi r}}{r} - \frac{{g_{\sigma NN}}^2}{4\pi} \frac{e^{-m_\sigma r}}{r} + \frac{{g^*_{\omega NN}}^2}{4\pi} \frac{e^{-m_v r}}{r} + \mathcal{O}(N_c^{-1})$$

Traditional approach

- Solve the Schrödinger's eq. with a regular BC $u_p(0) = 0$

$$-u_p''(r) + M_N\, V(r)\, u_p(r) = p^2\, u_p(r)$$

- Take the asymptotic condition for $r >> 1/m_\pi$

$$u_p(r) \to \frac{\sin\left(pr + \delta_0(p)\right)}{\sin \delta_0(p)}$$

- Unnaturally large scattering length $\alpha_0 = -23.74(2)\mathrm{fm}$.
- Any change in the potential has a dramatic effect,

$$\Delta\alpha_0 = \alpha_0^2 M_N \int_0^\infty \Delta V(r) u_0(r)^2 \mathrm{d}r$$

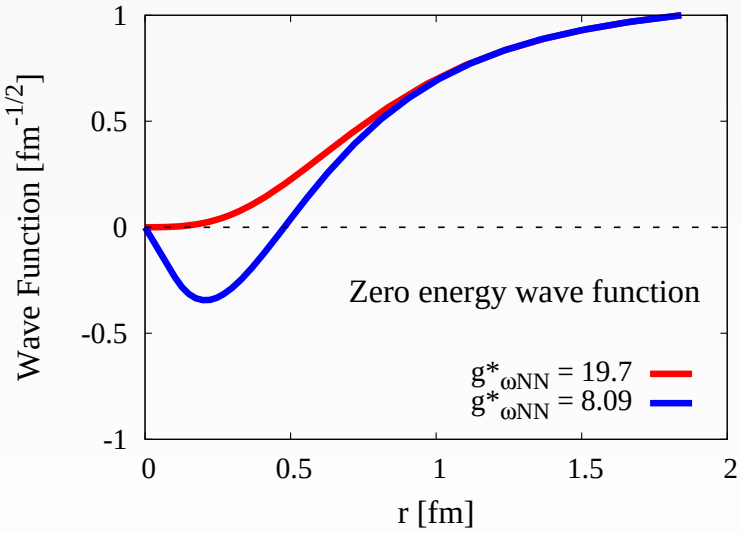
- Potential parameters must be fine tuned, in particular short distance physics !!

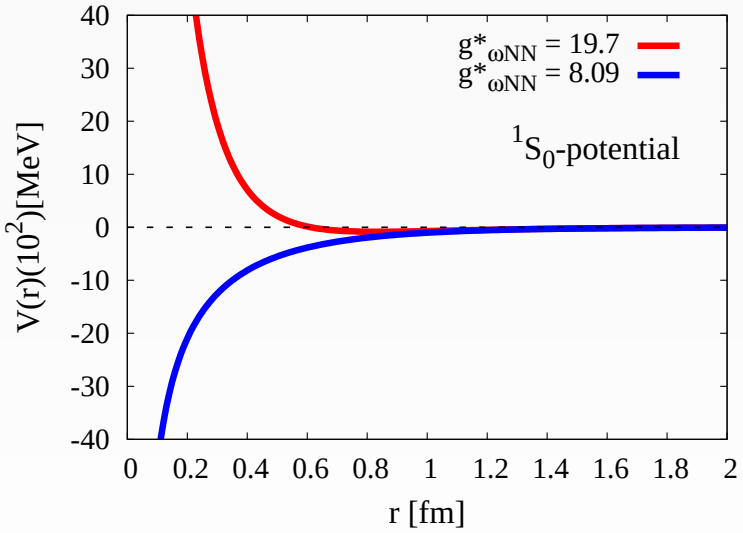
Fit of m_σ, $g_{\sigma NN}$ and $g^*_{\omega NN}$ to NN phase shifts

Two possible scenarios (very well determined)

$$\begin{aligned} m_\sigma &= 500.9(5)\text{MeV} \\ g_{\sigma NN} &= 9.61(1) \\ g^*_{\omega NN} &= 8.09(2) \\ \chi^2/DOF &= 0.484 \end{aligned}$$

$$\begin{aligned} m_\sigma &= 547.55(4)\text{MeV} \\ g_{\sigma NN} &= 13.559(8) \\ g^*_{\omega NN} &= 19.68(2) \\ \chi^2/DOF &= 0.869 \end{aligned}$$

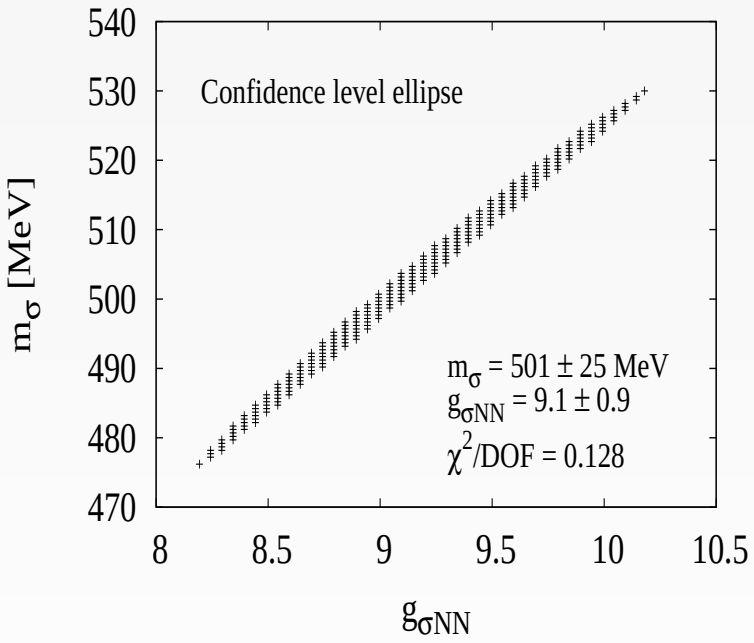


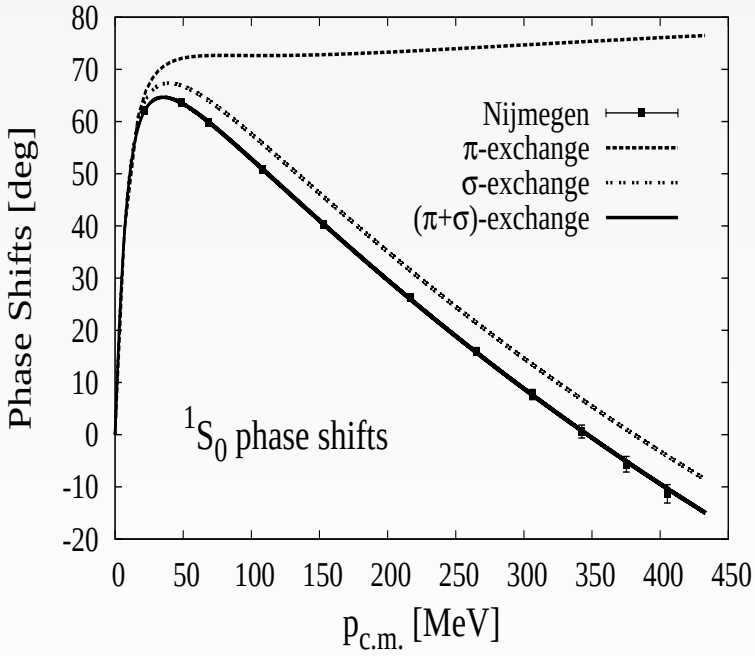


Natural values for couplings imply spurious (deeply) bound state

Renormalization

Short distance physics encoded in LEP's, fine tuning disappear





$$g_{\sigma NN} = 9.1(0.9),\ m_\sigma = 501(25) MeV,\ \chi^2/DOF = 0.128$$

Old Nuclear Physics Symmetries [Wigner $SU(4)$]

- Spin-isospin symmetry with 15-generators: T^a, S^i, G^{ai}
- Irreducible representations of $SU(4)$:
  One nucleon state, a quartet

$$\mathbf{4} = (p\uparrow, p\downarrow, n\uparrow, n\downarrow) = (S = 1/2, T = 1/2)$$

Two nucleon states, two supermultiplet $(-1)^{S+L+T} = -1$

$$\begin{aligned} \mathbf{6}_A &= (0,1) \oplus (1,0) \quad L = 0, 2, \ldots ({}^1S_0, {}^3S_1), ({}^1D_2, {}^3D_{1,2,3}), \ldots \\ \mathbf{10}_S &= (0,0) \oplus (1,1) \quad L = 1, 3, \ldots ({}^1P_1, {}^3P_{0,1,2}), ({}^1F_3, {}^3F_{2,3,4}), . \end{aligned}$$

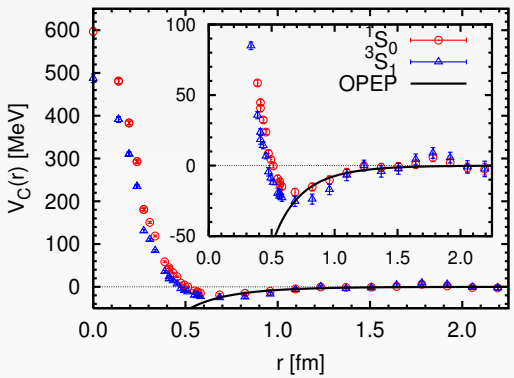
- Symmetry of the potential $\Rightarrow$ Symmetry of the S-matrix

$$V_{{}^1L}(r) = V_{{}^3L}(r) \Rightarrow \delta_{{}^1L}(p) = \delta_{{}^3L}(p)$$

$$\boxed{V_{{}^1S_0}(r) = V_{{}^3S_1}(r) \Rightarrow \delta_{{}^1S_0}(p) = \delta_{{}^3S_1}(p)}$$

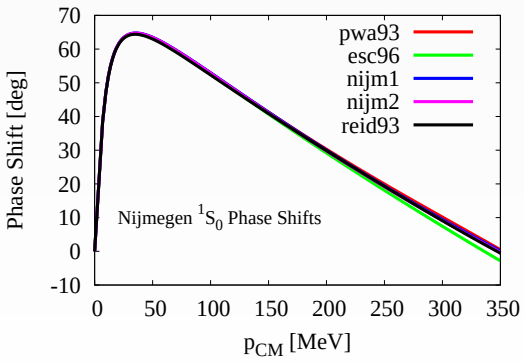
Puzzle: $V_{^1S_0}(r) = V_{^3S_1}(r)$ but $\delta_{^1S_0}(p) \neq \delta_{^3S_1}(p)$

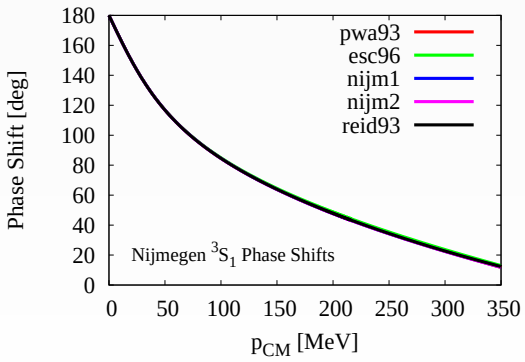
- Lattice QCD calculations (S. Aoki, T. Hatsuda, N. Ishii)



$$\Rightarrow V_{^1S_0}(r) \simeq V_{^3S_1}(r)$$

- S-wave phase shifts (Nijmegen group) $\rightarrow \delta_{^1S_0}(p) \neq \delta_{^3S_1}(p)$





Solution: Long Distances Symmetry

- Traditional approach:

$$\left.\begin{array}{l} V_s(r) = V_t(r) \\ u_s(0) = u_t(0) = 0 \end{array}\right\} \quad \Rightarrow \quad \left\{\begin{array}{l} \alpha_s = \alpha_t, \\ r_s = r_t, \\ \delta_s(p) = \delta_t(p) \end{array}\right.$$

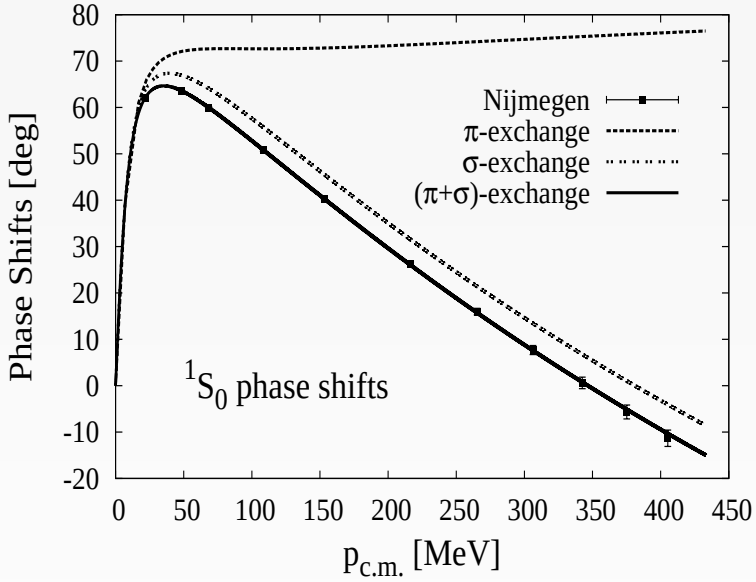
- Renormalization viewpoint:
  The symmetry is postulated at long distances but broken at short distances

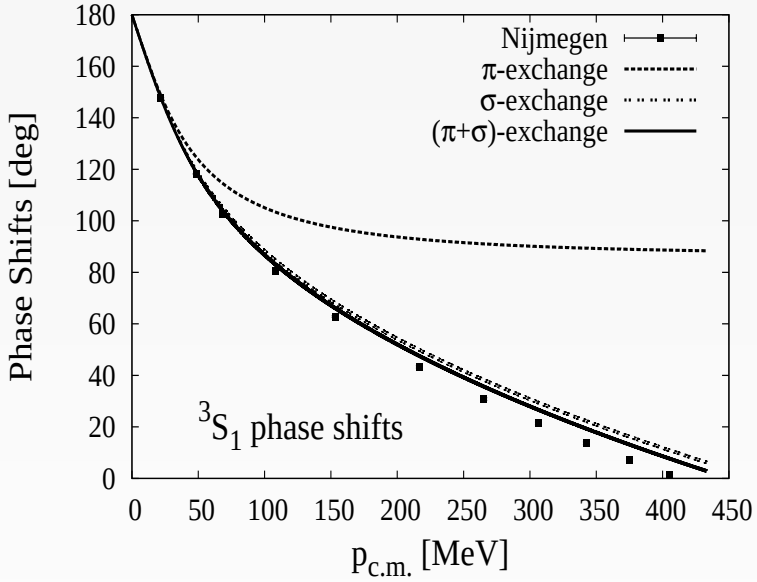
$$\left.\begin{array}{l} V_s(r)|_{r\to\infty} = V_t(r)|_{r\to\infty} \\ u_s(0^+) \neq u_t(0^+) \end{array}\right\} \quad \Rightarrow \quad \left\{\begin{array}{l} \alpha_s \neq \alpha_t, \\ r_s \neq r_t, \\ \delta_s(p) \neq \delta_t(p) \end{array}\right.$$

Long Distances Symmetry (phases connected)

A symmetry of the potential is not a symmetry of the S-matrix.

ACC + Ruiz Arriola, PRC **78**, 054002 (2008)





$$k\cot\delta_{^1S_0}(k) = \frac{\alpha_{^1S_0}\mathcal{A}(k)+\mathcal{B}(k)}{\alpha_{^1S_0}\mathcal{C}(k)+\mathcal{D}(k)} \quad , \quad k\cot\delta_{^3S_1}(k) = \frac{\alpha_{^3S_1}\mathcal{A}(k)+\mathcal{B}(k)}{\alpha_{^3S_1}\mathcal{C}(k)+\mathcal{D}(k)}$$

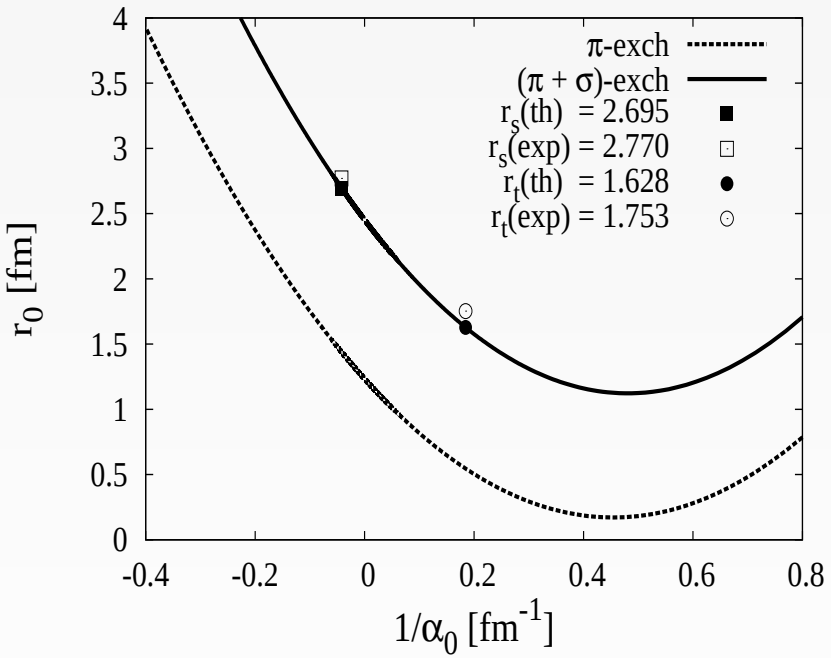
$$V_{^1S_0}(r) = V_{^3S_1}(r)$$

$$\alpha_{^1S_0} = -23.74 \text{ fm} \quad , \quad \alpha_{^3S_1} = 5.42 \text{ fm}$$

Long Distances Symmetry (Wigner correlation)

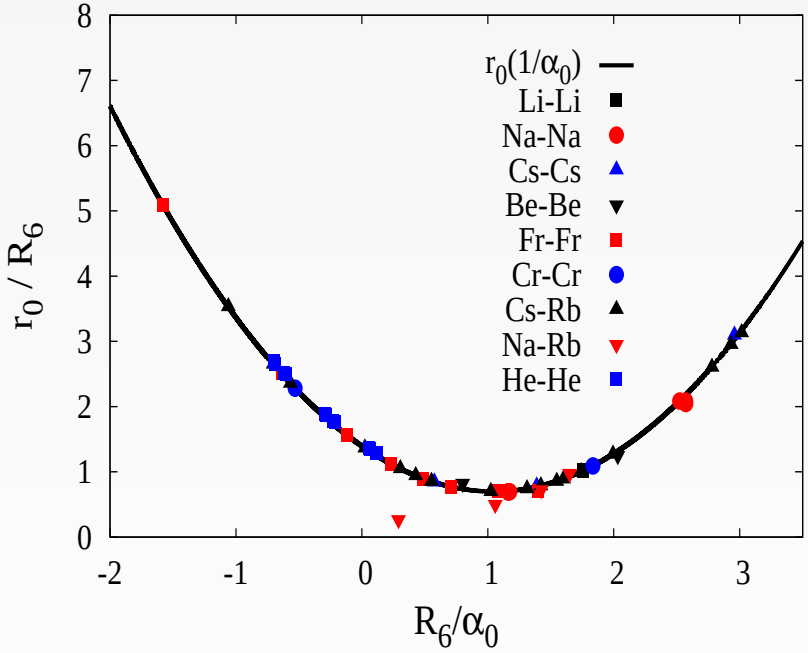
$r_0(\alpha_0)$ universal relation ($r_s \neq r_t$ because $\alpha_s \neq \alpha_t$)

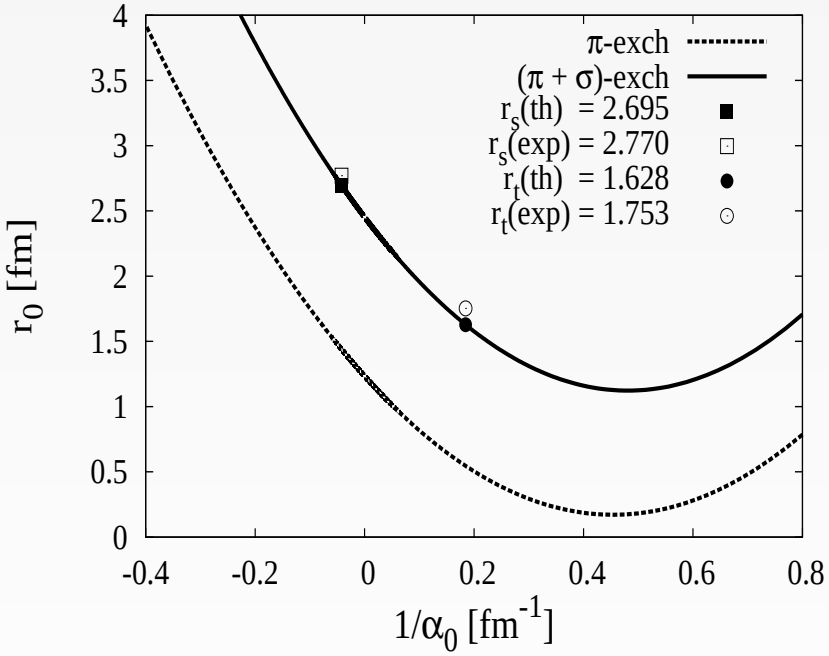
$$
\begin{aligned}
r_0 &= 1.3081 - \frac{4.5477}{\alpha_0} + \frac{5.1926}{\alpha_0^2} \quad (\pi) \\
&= 1.5089\ \text{fm}(\alpha_0 = \alpha_s)(\text{exp. } 2.770\text{fm}) \\
&= 0.6458\ \text{fm}(\alpha_0 = \alpha_t)(\text{exp. } 1.753\text{fm}) \\
r_0 &= 2.4567 - \frac{5.5284}{\alpha_0} + \frac{5.7398}{\alpha_0^2} \quad (\pi + \sigma) \\
&= 2.6989\ \text{fm}(\alpha_0 = \alpha_s)(\text{exp. } 2.770\text{fm}) \\
&= 1.6280\ \text{fm}(\alpha_0 = \alpha_t)(\text{exp. } 1.753\text{fm})
\end{aligned}
$$



Long Distances Symmetry (Wigner correlation)

$r_0(\alpha_0)$ universal relation (atomic physics analogous)





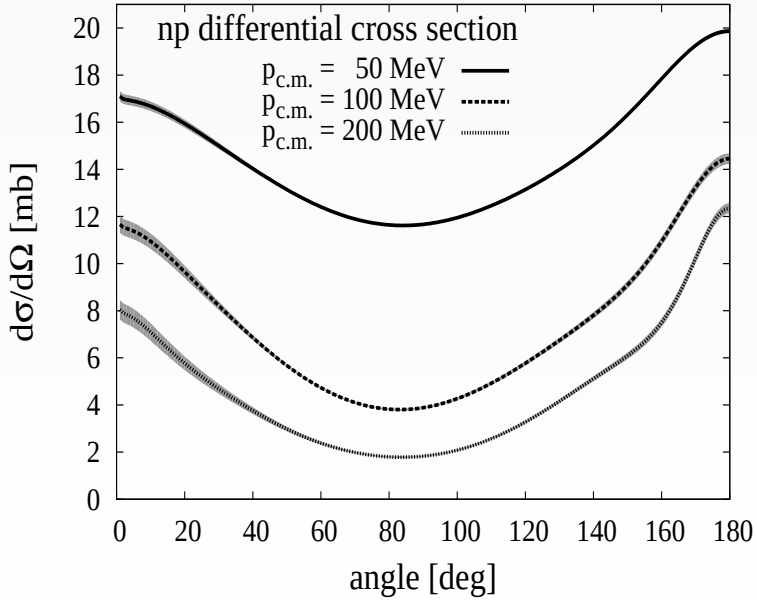
ACC + Ruiz Arriola, PRA **81**, 044701 (2010)

Old Nuclear Physics Symmetries [Serber]

- Saturation of nuclear forces: need singular Serber forces

$$V = \frac{1}{2}(1 + P_M)V(r)$$

- No interaction in odd L-waves !!
- Nature shows a clear Serber symmetry:



$$f_{np}(\theta) \simeq f_{np}(\theta - \pi)$$

Potential sum rules

Assume the potential to be central + (small) non-central

$$\begin{aligned} V_{NN} &= V_0 + V_1 \\ V_0 &= V_C + \tau W_C + \sigma V_S + \tau\sigma W_S, \\ V_1 &= (V_T + \tau W_T) S_{12} + (V_{LS} + \tau W_{LS}) L \cdot S, \end{aligned}$$

Define the mean potential (for the center of the multiplet)

$$V_{^{2S+1}L}(r) = \frac{\sum_{J=L-S}^{L+S} (2J+1) V_{^3L_J}(r)}{(2S+1)(2L+1)}$$

Serber requires vanishing odd-L waves

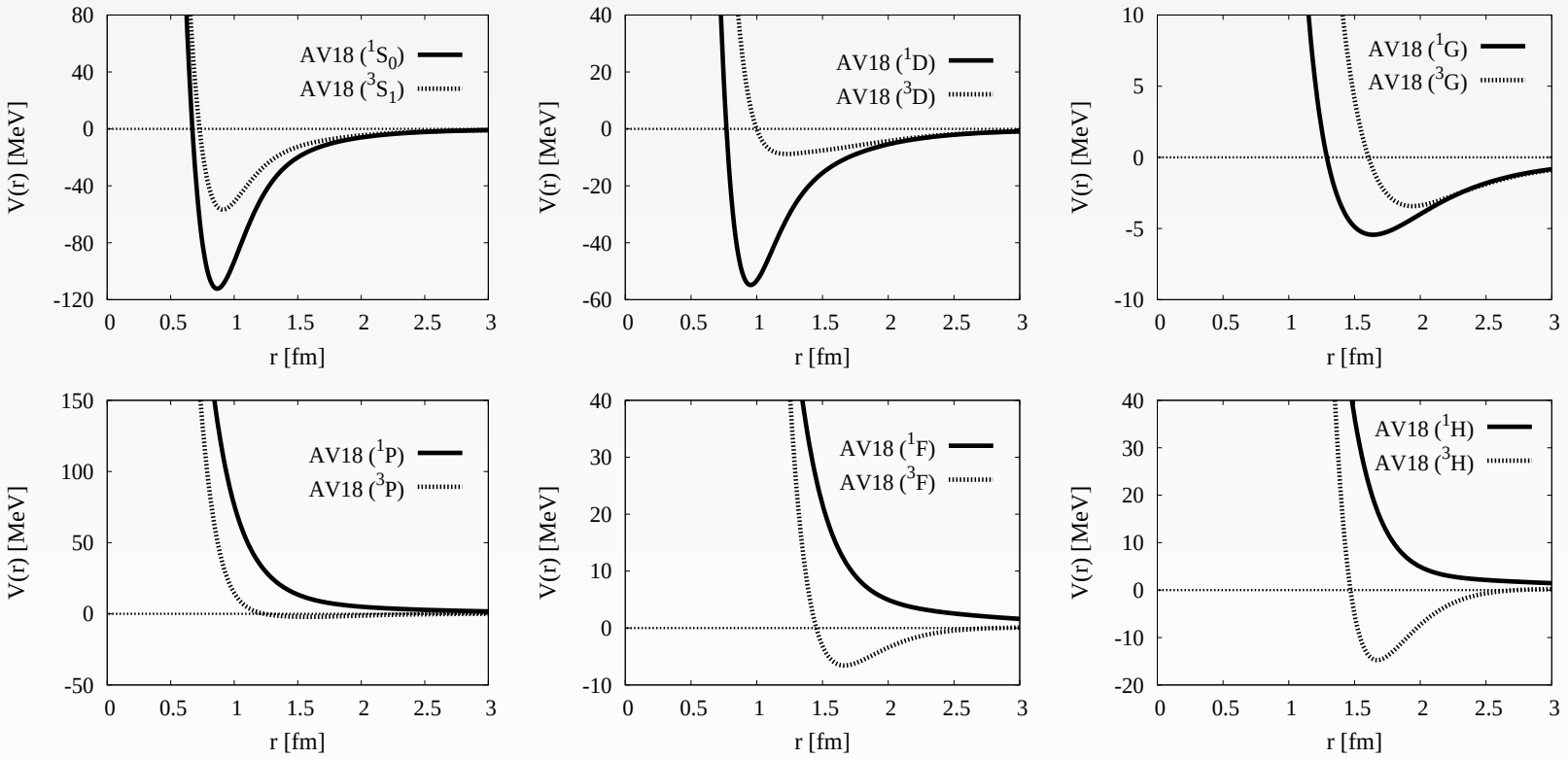
$$V_{^1L}(r) = V_{^3L}(r) = 0 \qquad \text{odd - L},$$

Wigner requires spin independence in all waves

$$V_{^1L}(r) = V_{^3L}(r) \qquad \text{all - L}$$

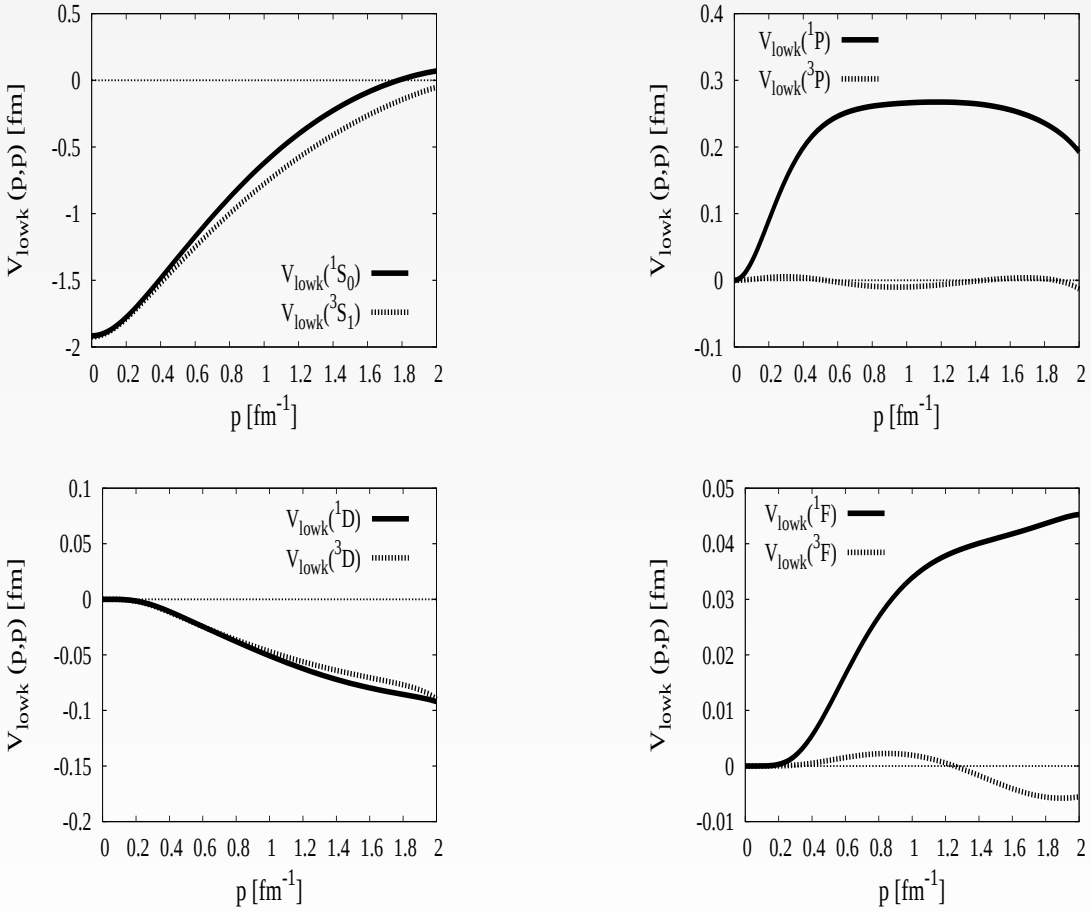
Wigner and Serber are incompatible !!

Potential sum rules (Argon V18)



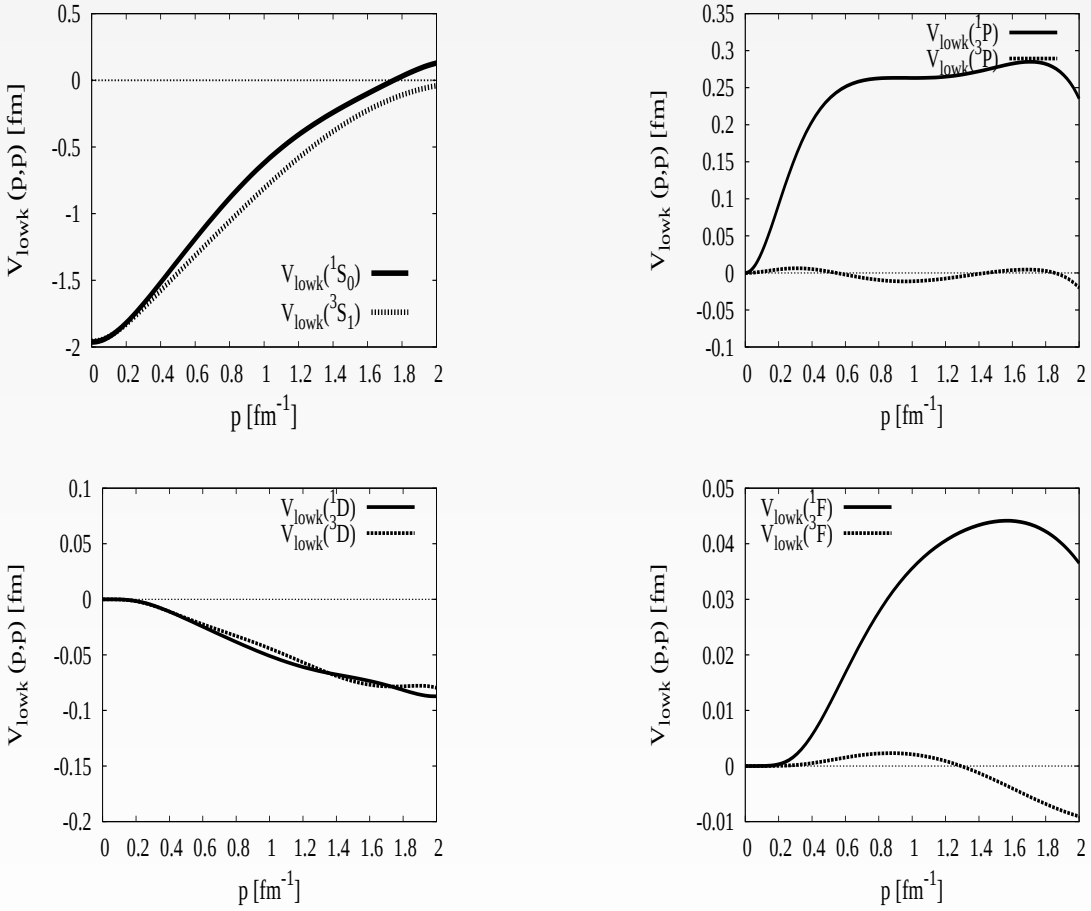
Even-L waves Wigner symmetry while odd-L triplet Serber symmetry

Potential sum rules (Argon V18 V_{lowk})



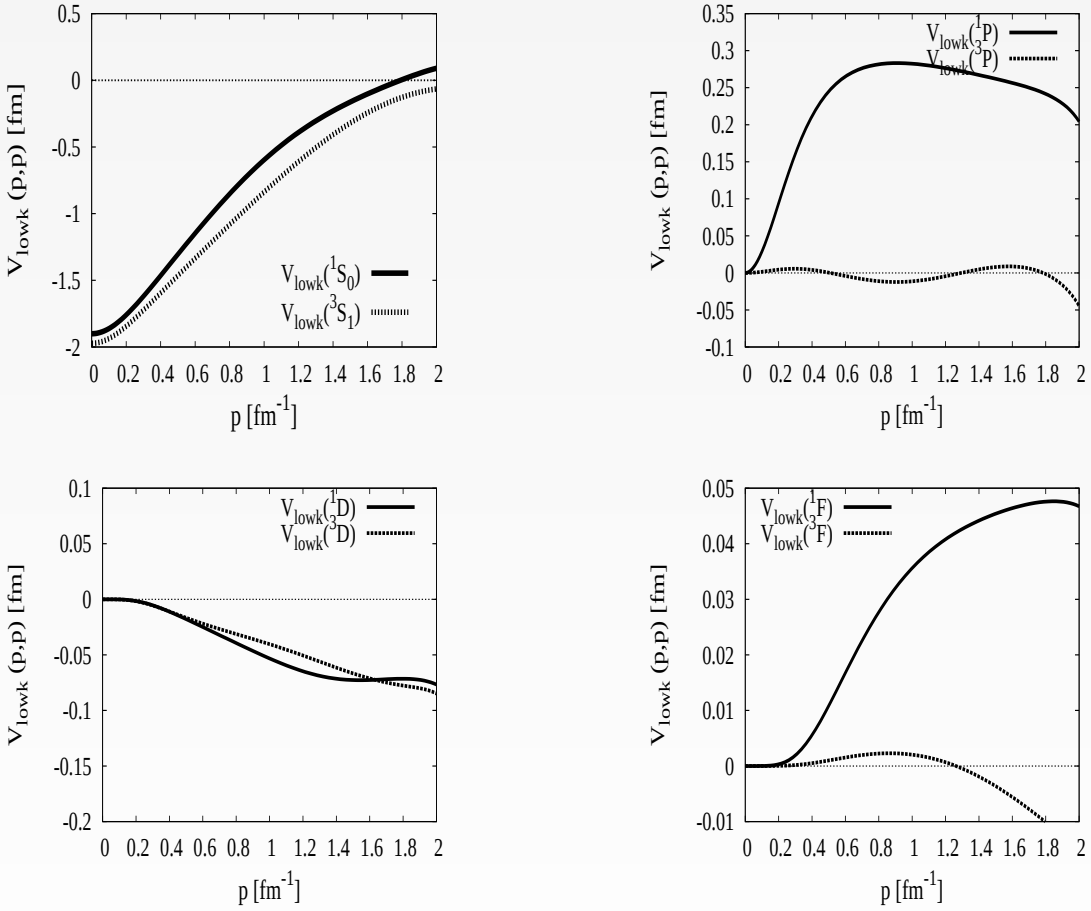
Even-L waves Wigner symmetry while odd-L triplet Serber symmetry

Potential sum rules ($\chi N^3LO\ V_{lowk}\ \Lambda = 500MeV$)



Even-L waves Wigner symmetry while odd-L triplet Serber symmetry

Potential sum rules ($\chi N^3LO\ V_{lowk}\ \Lambda = 600MeV$)



Even-L waves Wigner symmetry while odd-L triplet Serber symmetry

Non-central Phase Shifts Sum Rules

First order perturbation theory $\delta_{LJ}^{ST} = \delta_L^{ST} + \Delta\delta_{LJ}^{ST}$:

$$\delta_{^1P_1} = \frac{1}{9}\left(\delta_{^3P_0} + 3\delta_{^3P_1} + 5\delta_{^3P_2}\right) \qquad \text{Wigner}$$

$$= \delta_{^3P} = 0 \qquad \text{Serber}$$

$$\delta_{^1D_2} = \frac{1}{15}\left(3\delta_{^3D_1} + 5\delta_{^3D_2} + 7\delta_{^3D_3}\right)$$

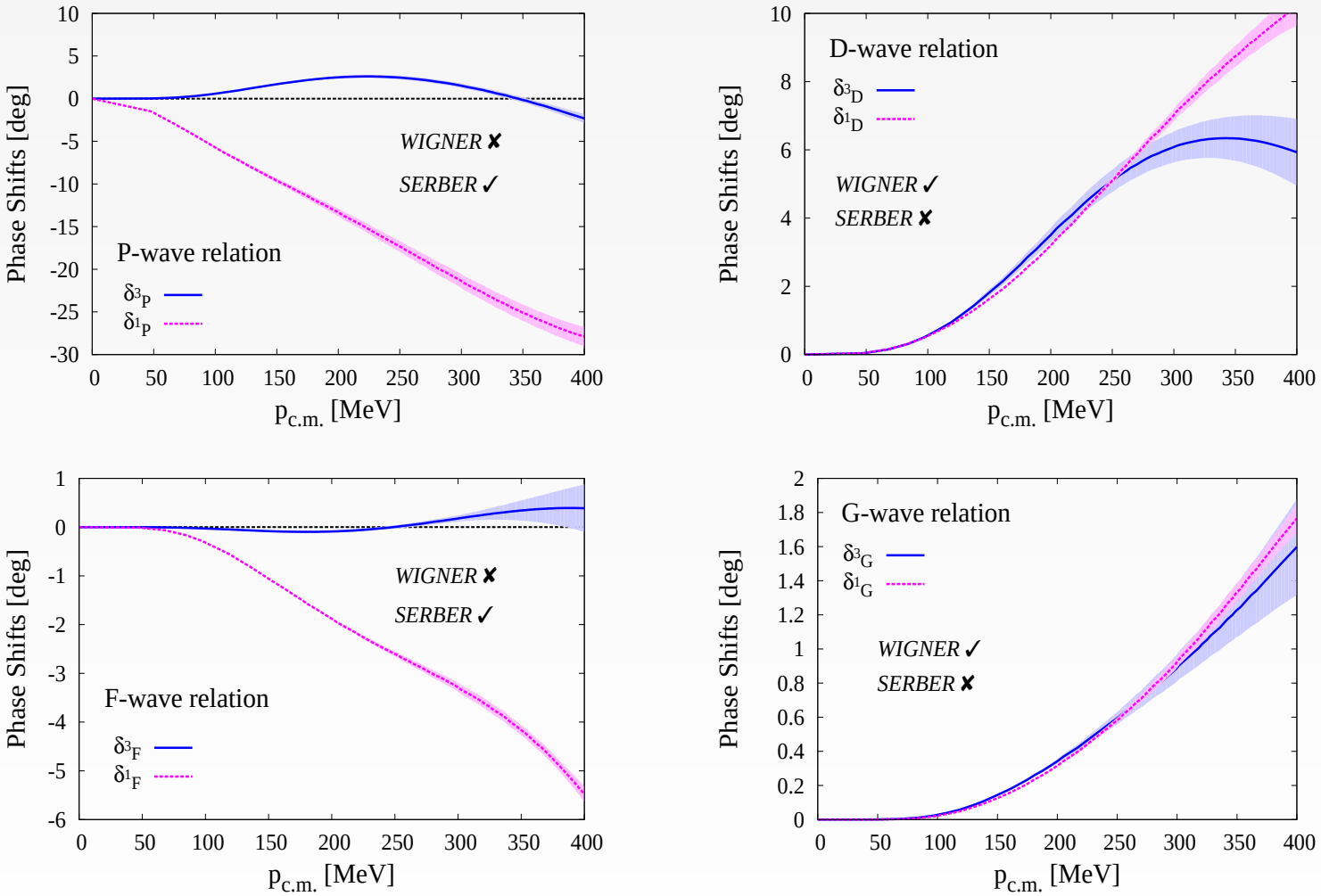
$$\delta_{^1F_3} = \frac{1}{21}\left(5\delta_{^3F_2} + 7\delta_{^3F_3} + 9\delta_{^3F_4}\right) \qquad \text{Wigner}$$

$$= \delta_{^3F} = 0 \qquad \text{Serber}$$

$$\delta_{^1G_4} = \frac{1}{27}\left(7\delta_{^3G_3} + 9\delta_{^3G_4} + 11\delta_{^3G_5}\right)$$

Wigner and Serber are incompatible !!

Nijmegen Phase Shift Sum Rules



Even-L waves Wigner symmetry while odd-L triplet Serber symmetry

Large N_c and Wigner-Serber Symmetries

- Large N_c *implies* Wigner symmetry in even-L channels

$$V_{^1L} = V_{^3L} = V_C(r) - 3W_S(r) + \mathcal{O}(N_c^{-1}), \quad \text{even-L}$$

- Large N_c *allows* Wigner violation in odd-L and Serber violation in spin singlet ^1L channels

$$\begin{aligned} V_{^1L} &= V_C(r) + 9W_S(r) + \mathcal{O}(1/N_c), \quad \text{odd-L} \\ V_{^3L} &= V_C(r) + \ \ W_S(r) + \mathcal{O}(1/N_c), \quad \text{odd-L} \end{aligned}$$

Symmetry breaking is compatible with large N_c !!!

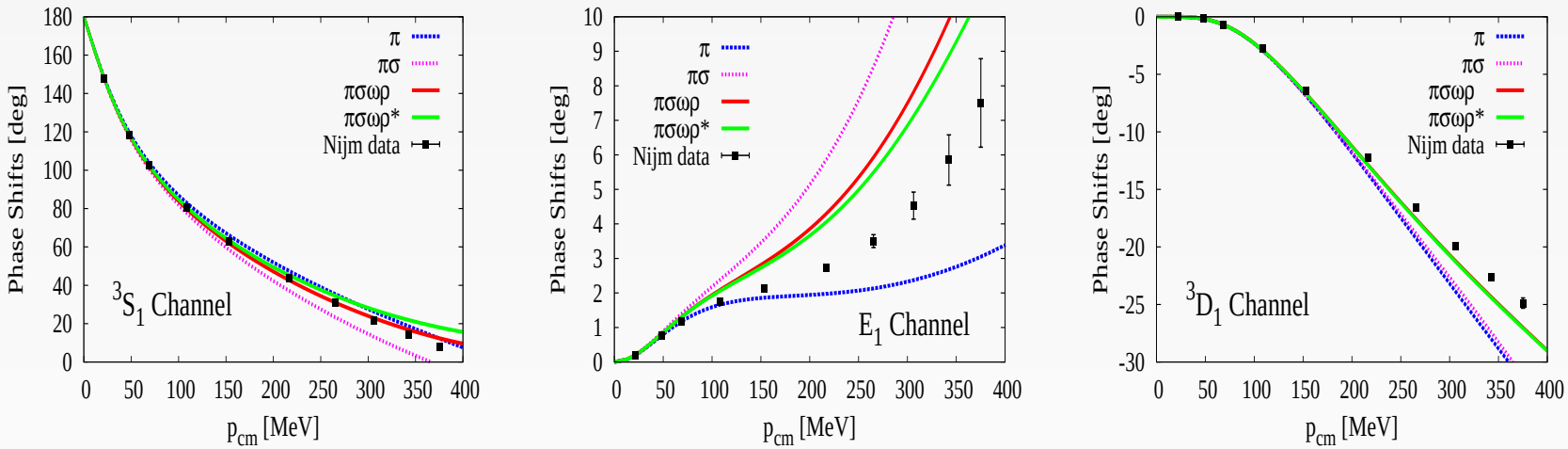
- Large N_c may explain Serber symmetry triplet ^3L channels if $W_S(r) = -V_C(r)$ which is fulfilled with the identification $m_\sigma = m_\rho = m_\omega$

$$\begin{aligned} V_C(r) &= -\frac{g^2_{\sigma NN}}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g^2_{\omega NN}}{4\pi}\frac{e^{-m_\omega r}}{r} \\ W_S(r) &= \frac{1}{12}\frac{g^2_{\pi NN}}{4\pi}\frac{m^2_\pi}{\Lambda^2_N}\frac{e^{-m_\pi r}}{r} + \frac{1}{6}\frac{f^2_{\rho NN}}{4\pi}\frac{m^2_\rho}{\Lambda^2_N}\frac{e^{-m_\rho r}}{r} \end{aligned}$$

ACC + Ruiz Arriola, PRC **80**, 014002 (2009)

Renormalized Deuteron (properties and phases)

Take natural values for couplings and make model independent predictions

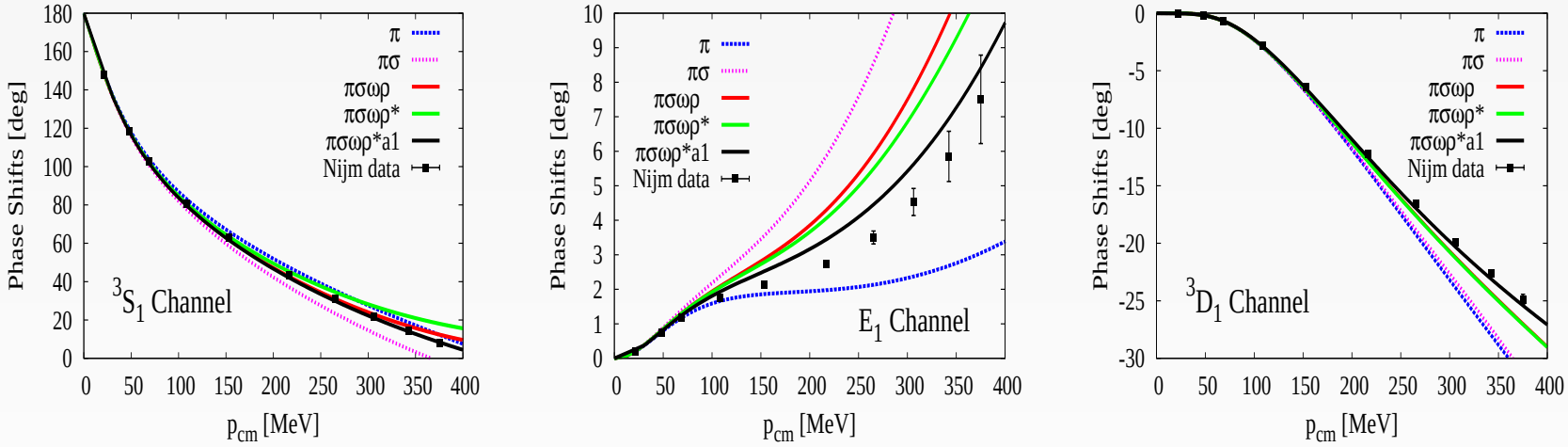


	γ(fm^{-1})	η	A_S(fm$^{-1/2}$)	r_m(fm)	Q_d(fm^2)	P_D	$\langle r^{-1}\rangle$	α_0(fm)	α_{02}(fm^3)	α_2(fm^5)	r_0(fm)
π	Input	0.02633	0.8681	1.9351	0.2762	7.88%	0.476	5.335	1.673	6.169	1.638
$\pi\sigma$	Input	0.02599	0.9054	2.0098	0.2910	6.23%	0.432	5.335	1.673	6.169	1.638
$\pi\sigma\rho\omega$	Input	0.02597	0.8902	1.9773	0.2819	7.22%	0.491	5.444	1.745	6.679	1.788
$\pi\sigma\rho\omega^*$	Input	0.02625	0.8846	1.9659	0.2821	9.09%	0.497	5.415	1.746	6.709	1.748
NijmII	Input	0.02521	0.8845(8)	1.9675	0.2707	5.635%	0.4502	5.418	1.647	6.505	1.753
Reid93	Input	0.02514	0.8845(8)	1.9686	0.2703	5.699%	0.4515	5.422	1.645	6.453	1.755
Exp. [1]	0.231605	0.0256(4)	0.8846(9)	1.9754(9)	0.2859(3)	5.67(4)		5.419(7)			1.753(8)

ACC + Ruiz Arriola, PRC **81**, 044002 (2010)

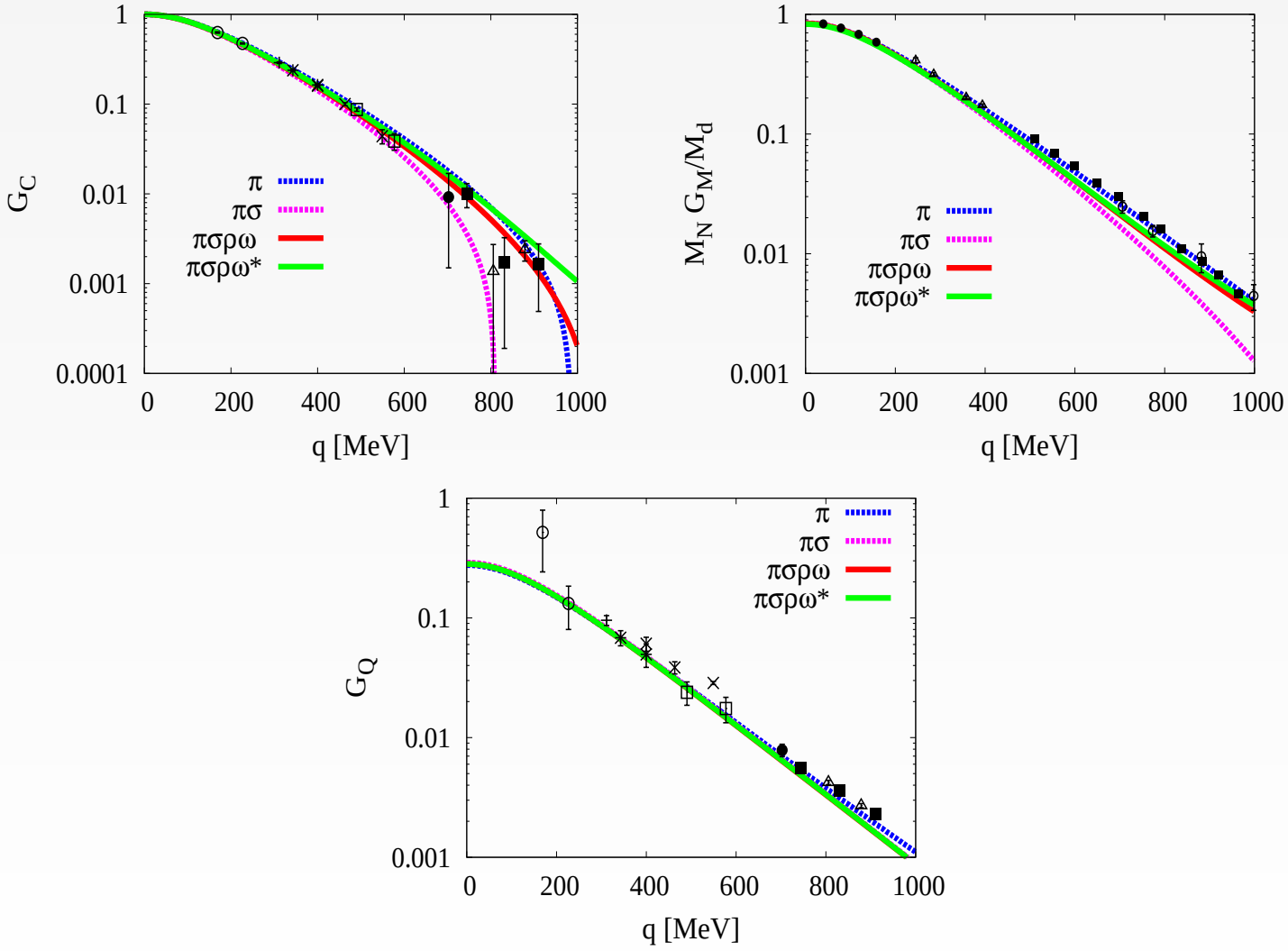
Renormalized Deuteron (properties and phases)

We can also include the axial meson (a_1) which is allowed in the large N_c
$m_{a_1} = \sqrt{2} m_\rho$ (VMD), $g_{a_1 NN} = m_{a_1}/m_\pi f_{\pi NN}$ (Schwinger relation)

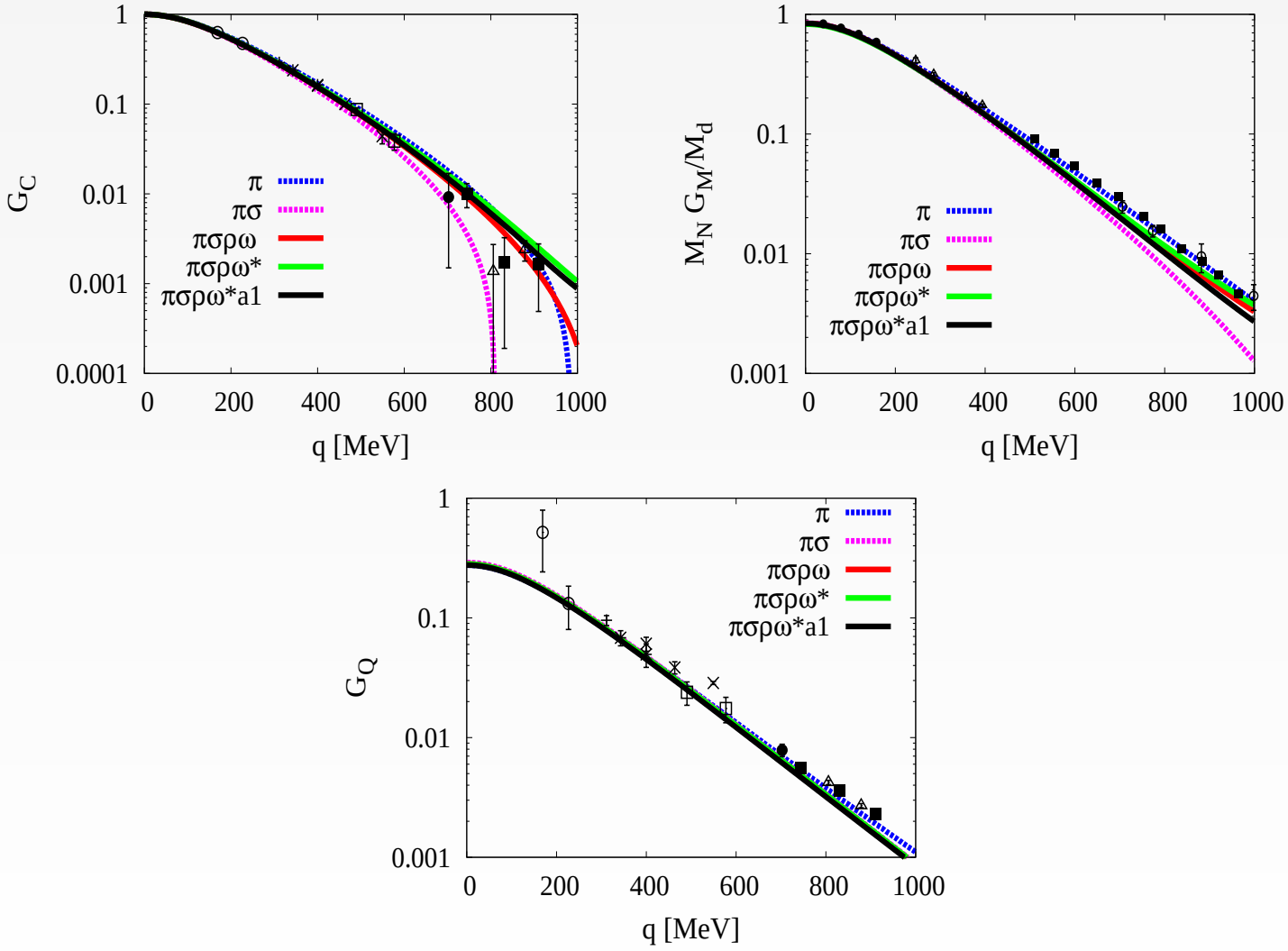


	γ(fm^{-1})	η	A_S(fm$^{-1/2}$)	r_m(fm)	Q_d(fm^2)	P_D	$\langle r^{-1}\rangle$	α_0(fm)	α_{02}(fm^3)	α_2(fm^5)	r_0(fm)
π	Input	0.02633	0.8681	1.9351	0.2762	7.88%	0.476	5.335	1.673	6.169	1.638
$\pi\sigma$	Input	0.02599	0.9054	2.0098	0.2910	6.23%	0.432	5.335	1.673	6.169	1.638
$\pi\sigma\rho\omega$	Input	0.02597	0.8902	1.9773	0.2819	7.22%	0.491	5.444	1.745	6.679	1.788
$\pi\sigma\rho\omega^*$	Input	0.02625	0.8846	1.9659	0.2821	9.09%	0.497	5.415	1.746	6.709	1.748
$\pi\sigma\rho\omega^* a_1$	Input	0.02549	0.8985	1.9953	0.2810	5.84%	0.463	5.487	1.735	6.624	1.849
NijmII	Input	0.02521	0.8845(8)	1.9675	0.2707	5.635%	0.4502	5.418	1.647	6.505	1.753
Reid93	Input	0.02514	0.8845(8)	1.9686	0.2703	5.699%	0.4515	5.422	1.645	6.453	1.755
Exp. [1]	0.231605	0.0256(4)	0.8846(9)	1.9754(9)	0.2859(3)	5.67(4)		5.419(7)			1.753(8)

Renormalized Deuteron (EM Form Factors IA)



Renormalized Deuteron (EM Form Factors IA)

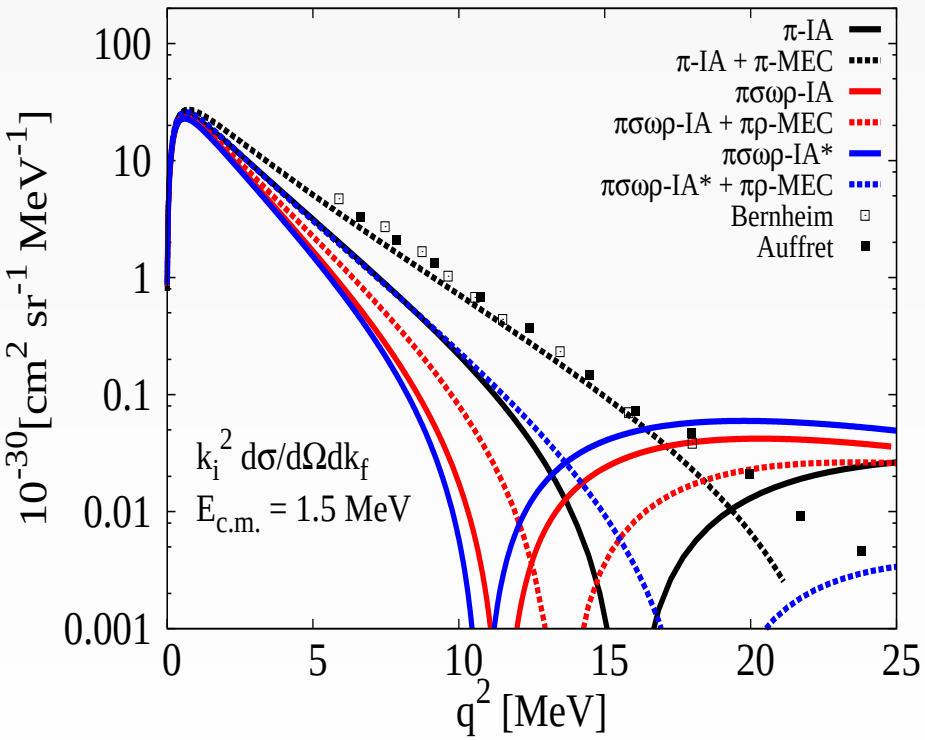


Backward deuteron electro-disintegration

$$e^- + d\ (^3S_1 - ^3D_1) \rightarrow e^- + np\ (^1S_0)$$ [Adler,Hockert,Riska]

$$\frac{d\sigma}{dE_f d\Omega}(180^0) = \frac{\alpha^2}{4\pi}\ \frac{pq^2}{E_i^2 M_N}\ [g(q) + h(q)]^2$$

$p =$ np c.m. momentum,
$E_f =$ final e^- energy,
$E_i =$ incident e^- energy,
$M_N =$ nucleon mass,
$q =$ momentum transfer,
$g(q), h(q) =$ structure functions
(IA + π-MEC + ρ-MEC)



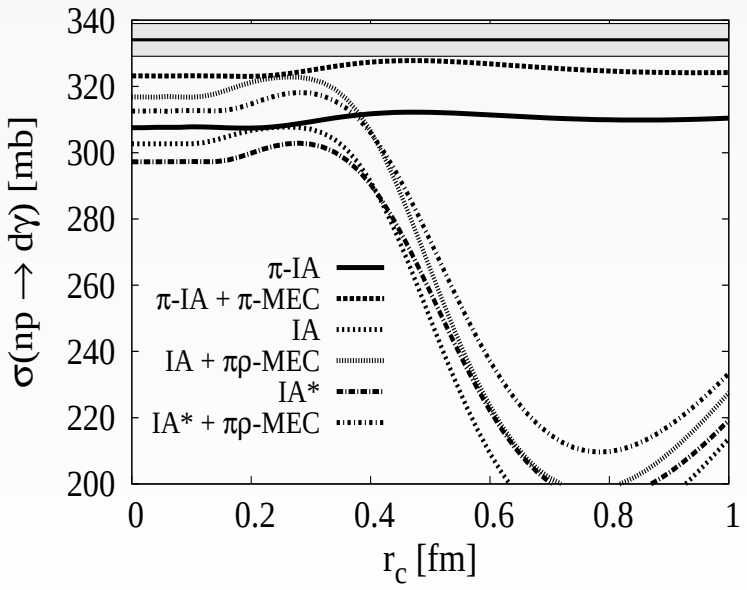
Neutron capture ($n + p \to \gamma + d$)

$$n + p\ (^1S_0) \to \gamma + d\ (^3S_1 -^3 D_1)$$ [Adler,Hockert,Riska]

$$\sigma = \frac{\pi\alpha\omega^3}{2pM_N}\,[g(0) + h(0)]^2$$

Contribution	$\sigma(np \to d\gamma)$ [mb]
$\pi\sigma\rho\omega$ IA	302.7
$\pi\sigma\rho\omega^*$ IA	297.3
π-exch + π-MEC	323.1
$\pi\sigma\rho\omega$-exch + $\pi\rho$-MEC	317
$\pi\sigma\rho\omega^*$-exch + $\pi\rho$-MEC	312.5
Experimental	334.2 (5)

$$\omega = B_d = 2.2MeV$$
$$p = 3.4451 \times 10^{-3} MeV$$



Outline of the talk

SUMMARY AND CONCLUSIONS

- We have analyzed the NN interaction from a different approach which is the $1/N_c$ expasion of QCD. This potential need to be renormalized.
- Although still incomplete, since tower of mesons and the Δ-isobar should be include consistently, results for central waves and the deuteron are encouraging.
- There is a CHOICE between short distance fine-tuning and renormalization.
- Our point of view: minimize the impact of things you know worst.

$\Rightarrow$ Short distance physics hardly accessible $\Leftarrow$

- You may learn a lot more admitting fixing fine-tuned parameters as independent variables on their own.

$\Rightarrow$ For example the scattering length α_0 $\Leftarrow$

- Once this is done you obtain short distance insensitivity and you can answer about accessible long distance issues.